A6
Dimensional Analysis

DEARBORN | SCIENCE LEARNING CENTER

Objective

- Understand the definition of units
- Add, subtract, multiply and divide numbers with identical units
- Convert one unit to another
- Multiply given information by conversion factors

Introduction

Most science courses require solving problems through mathematical operations. One of the most useful approaches to solving these problems is Dimensional Analysis.

This approach clearly:

- Identifies the “units” or “dimensions” on the numbers used in a mathematical operation
- Indicates how the numbers in the problem should be combined (add, subtract, multiply, or divide)
- Demonstrates whether the answer obtained is in the form requested by the problem

Definition of Units

A unit is "a determinate quantity (such as length, time, heat, or value) adopted as a standard of measurement." Unit defines what a number is while a number defines the magnitude of the unit.

Suppose you were asked "Do you have 5?" You would probably ask in return, "Five what?" It might be five cents, five dollars, five fingers, or even five pounds of salt. The number by itself is meaningless.

You have to say the quantity of *what* you are talking about and include the unit (or dimension) of the number.

In the above, it is the cents, dollars, fingers, or pounds which are the units.

Mathematical Manipulation of Units

For multiplication and division, units are treated in the same manner as numbers. Units are multiplied or divided just as numbers are multiplied or divided.

For example:

$4\ \text{cm} \times 2\ \text{g} = 8\ \text{cm} \cdot \text{g}$

$2\ \text{cm} \times 3\ \text{cm} = 6\ \text{cm}^2$

$$\frac{8\ \text{mm}}{3\ \text{s}} = 2.7\ \frac{\text{mm}}{\text{s}}$$

Mathematical Manipulation of Units

A unit in the denominator can be placed in the numerator if the sign of the superscript is changed:

$$\frac{2\ \text{g}}{\text{cm}} \text{ can also be written as } 2\ \text{g} \cdot \text{cm}^{-1}$$

More examples:

$$\frac{1}{2\ \text{cm}} = \frac{1\ \text{cm}^{-1}}{2}$$

$$\frac{3\ \text{g}}{4\ \text{cm}^2} = \frac{3\ \text{g} \cdot \text{cm}^{-2}}{4}$$

$$\frac{1\ \text{g}}{3\ \text{m}^{-2}} = \frac{1\ \text{g} \cdot \text{m}^2}{3}$$

Mathematical Manipulation of Units

In chemistry, we use the ideal gas constant R where,

$$R = \frac{8.314\ \mathrm{J}}{(\mathrm{mol} \cdot \mathrm{K})}$$

More ways to write it:

$$R = \frac{8.314}{\mathrm{J}^{-1} \cdot \mathrm{mol} \cdot \mathrm{K}}$$

$$R = \frac{8.314\ \mathrm{mol}^{-1} \cdot \mathrm{K}^{-1}}{\mathrm{J}^{-1}}$$

$$R = 8.314\ \mathrm{J} \cdot \mathrm{mol}^{-1} \cdot \mathrm{K}^{-1}$$

$$R = \frac{8.314\ \mathrm{J} \cdot \mathrm{mol}^{-1}}{\mathrm{K}}$$

Section I Problems

Below are several practice problems. Work them out on a piece of scratch paper. The answers are at the end of the module.

1) $4\ \text{cm} \times 3\ \text{g} \cdot \text{cm} = ?$

2) $2\ \text{cm} \cdot \text{s}^{-1} \times 3\ \text{g} \cdot \text{s}^{2} = ?$

3) $\dfrac{3\ \text{g} \cdot \text{cm} \cdot \text{s}^{-1} \times 4\ \text{cm} \cdot \text{s}^{-1}}{2\ \text{g} \cdot \text{cm}^{-3}} = ?$

4) $\dfrac{2\ \text{g} \times 3\ \text{cm}^{2} \times 5\ \text{g} \cdot \text{cm}^{-3}}{3\ \text{s} \times 1\ \text{cm} \cdot \text{s}^{-1} \times 2\ \text{g} \cdot \text{cm}^{-2}} = ?$

Conversion Factors

When changing from one unit to another for the same quantity (length to length, cm to mm), a conversion factor is used.

Let's start with a simple example of a conversion factor. We all know that there are 24 hours in one day. This really means that 24 hours equal one day. Mathematically this is:

$$24 \text{ hours} = 1 \text{ day} \quad \text{or} \quad \frac{24 \text{ hours}}{1 \text{ day}}$$

You are probably more accustomed to saying "There are 24 hours per day." The "per" means "divided by".

Conversion Factors

Note that all conversion factors are equal to 1.

Take 60 seconds = 1 minute and divide both sides by 1 minute:

$$\frac{60 \text{ seconds}}{1 \text{ minute}} = \frac{1 \text{ minute}}{1 \text{ minute}}$$

$$\frac{1 \text{ minute}}{1 \text{ minute}} = 1 \quad \text{therefore} \quad \frac{60 \text{ seconds}}{1 \text{ minute}} = 1$$

If we take 60 seconds = 1 minute and divide both sides by 60 seconds:

$$\frac{1 \text{ minute}}{60 \text{ seconds}} \text{ also equals } 1$$

Conversion Factors

Here are several other examples:

$$24 \text{ hrs} = 1 \text{ day} \quad \text{or} \quad \frac{24 \text{ hrs}}{1 \text{ day}} \quad \text{or} \quad 24 \text{ hrs} \cdot \text{days}^{-1}$$

$$100 \text{ cents} = \$1 \quad \text{or} \quad \frac{100 \text{ cents}}{\$1} \quad \text{or} \quad 100 \text{ cents} \cdot \$^{-1}$$

$$2.54 \text{ cm} = 1 \text{ in} \quad \text{or} \quad \frac{2.54 \text{ cm}}{1 \text{ in}} \quad \text{or} \quad 2.54 \text{ cm} \cdot \text{in}^{-1}$$

Section II Problems

Here are more practice problems. For each problem, write the conversion factor and its reciprocal in terms of per single unit. The answers are at the end of the module.

5) There are 12 inches in 1 foot.

6) My car used 9.0 gallons of gasoline to travel 270 miles.

7) He earned $24 in 6.0 hours of work.

Solving Problems

Now let's turn our attention to problem solving by dimensional analysis.

1. All problems ask a question, and the answer should include a number and its dimensions.
2. All problems have some information given, which will also be a number and its dimensions.
3. You will multiply the information given by conversion factors, so that you cancel all dimensions that your final answer doesn’t need.
4. You may multiply by as many conversion factors as you wish since all conversion factors equal 1, and multiplying something by 1 does not change its value.

Solving Problems

Let's look at an example.

How many seconds are in 35 minutes?

- The question asked is "How many seconds?" Therefore, when you get an answer, it will have the dimension (seconds). The information given in the problem is 35 minutes. So we can start by writing down the dimension of the answer, then we supply the information given.

$$? \text{ s} = 35 \text{ min}$$

Solving Problems

We need to multiply by a conversion factor, that has the dimension (min) in the denominator. This will allow us to cancel the min in the information given. That way, our final answer will have the dimensions (s) in the numerator. Here, we will use the conversion factor $\frac{60\ \text{s}}{1\ \text{min}}$

$$?\ \text{s} = 35\ \cancel{\text{min}} \times 60\ \frac{\text{s}}{\cancel{\text{min}}}$$

$$= 35 \times 60\ \text{s}$$

$$= 2{,}100\ \text{s}$$

$$\boxed{2100\ \text{s} = 35\ \text{min}}$$

Solving Problems

Note: It matters which way you write the conversation factor.

Below shows the improper way.

How many seconds are in 35 minutes?

$$?\,\mathrm{s} = 35\,\mathrm{min} \times \frac{1\,\mathrm{min}}{60\,\mathrm{s}}$$

$$= \frac{35}{60}\,\mathrm{min}^2 \cdot \mathrm{s}^{-1}$$

$$= 0.58\,\mathrm{min}^2 \cdot \mathrm{s}^{-1}$$

Instead of cancelling out the units, we are ending up with more.

$$2100\,\mathrm{s} \neq 0.58\,\mathrm{min}^2 \cdot \mathrm{s}^{-1}$$

Solving Problems

Multiplying by only one conversion factor usually will not get you all the way to the dimensions of the answer. However, since all conversion factors equal one, you can multiply by as many as you need.

Q) How many hours are in six years?

- The question asked is "How many years?" The dimension of the answer, therefore, will be hrs. The information given is 6 years.

$$? \text{ hrs} = 6 \cancel{\text{years}} \times \frac{365 \cancel{\text{days}}}{1 \cancel{\text{year}}} \times \frac{24 \text{ hrs}}{1 \cancel{\text{day}}}$$

$$= 52{,}560 \text{ hrs}$$

$$\boxed{6 \text{ years} = 52{,}560 \text{ hrs}}$$

Solving Problems

Example 1:

How many miles in 2,640 yards?

Information given:

? miles = 2,640 yards

Conversion factors:

3 ft = 1 yard

5,280 ft = 1 mile

Solutions in the back of the module

Solving Problems

Example 2:

What is the cost of three shirts, if a box containing 12 shirts costs \$27?

Information given:

$$\$\,? = 3 \text{ shirts}$$

Conversion factors:

$$12 \text{ shirts} = \$27$$

Solutions in the back of the module

Solving Problems

Example 3:

What is the gas consumption in miles per gallon of an automobile if it uses 0.1 gallons of gas in 100 s when traveling 60 miles/hr?

Information given:

$$? \text{ miles} = 1 \text{ gallon}$$

Conversion factors:

$$0.1 \text{ gallons} = 100 \text{ s}$$

$$60 \text{ miles} = 1 \text{ hour}$$

Solutions in the back of the module

Solving Problems

Example 4:

How many grams HCl are in 3.800 mol HCl? The molar mass of HCl is 36.46 g/mol.

Information given:

$$? \text{ g HCl} = 3.800 \text{ mol HCl}$$

Conversion factors:

$$1 \text{ mol HCl} = 36.46 \text{ g/mol}$$

Solutions in the back of the module

Solving Problems

Example 5:

How many moles NH_3 are in 5.280 g NH_3? (The molar mass of NH_3 is 17.031 g/mol.)

Information given:

$$? \text{ mol } NH_3 = 5.280 \text{ g } NH_3$$

Conversion factors:

$$1 \text{ mol } NH_3 = 17.031 \text{ g/mol } NH_3$$

Solutions in the back of the module

Section III Problems

Here are several practice problems. The answers are at the end of the module.

8) How many seconds in 800 minutes?

9) How many dozens of eggs are there in 3,500 eggs?

10) How many moles of CO_2 are there in 360 g CO_2? (Molar mass of CO_2 is 44.01 g/mol)

11) How many grams of OH are in 0.4300 moles OH? (Molar mass of OH is 17.008 g/mol)

Mathematical Manipulation of Units

In mathematical manipulations, units are treated in the same manner as numbers. For <u>addition and subtraction</u>, the old saying that you can't add apples to oranges applies. You can only add or subtract numbers with identical units.

Example:

$$4 \text{ ft} + 5 \text{ ft} = 9 \text{ ft}$$

Both units are units of length, so the numbers can be added.

Mathematical Manipulation of Units

- $4\ \text{ft} + 5\ \text{ft}^2 = ?$

Since ft is a unit of length and ft^2, a unit of area, so their addition cannot be more simplified.

- $4\ \text{ft} + 5\ \text{in} = ?$

Both are units of length, but since they are different, they cannot be added until one is converted to the same unit as the other one.

If you have any questions, feel free to ask the assistant in the Science Learning Center for help. If you feel confident in solving these problems, ask the assistant for the posttest.

Section I Solutions

1) $4\ \mathrm{cm} \times 3\ \mathrm{g} \cdot \mathrm{cm} = \boxed{12\ \mathrm{g} \cdot \mathrm{cm}^2}$

2) $2\ \mathrm{cm} \cdot \mathrm{s}^{-1} \times 3\ \mathrm{g} \cdot \mathrm{s}^2 = \boxed{6\ \mathrm{cm} \cdot \mathrm{g} \cdot \mathrm{s}}$

3) $$\frac{3\ \mathrm{g} \cdot \mathrm{cm} \cdot \mathrm{s}^{-1} \times 4\ \mathrm{cm} \cdot \mathrm{s}^{-1}}{2\ \mathrm{g} \cdot \mathrm{cm}^{-3}} = \boxed{6\ \mathrm{cm}^5 \cdot \mathrm{s}^{-2}}$$

4) $$\frac{2\ \mathrm{g} \times 3\ \mathrm{cm}^2 \times 5\ \mathrm{g} \cdot \mathrm{cm}^{-3}}{3\ \mathrm{s} \times 1\ \mathrm{cm} \cdot \mathrm{s}^{-1} \times 2\ \mathrm{g} \cdot \mathrm{cm}^{-2}} = \boxed{5\ \mathrm{g}}$$

Section II Solutions

5) $12 \text{ inches} = 1 \text{ foot}$ or $\frac{12 \text{ inches}}{1 \text{ foot}}$ or $\frac{1 \text{ foot}}{12 \text{ inches}}$

6) $9.0 \text{ gallons} = 270 \text{ miles}$ or $\frac{9.0 \text{ gallons}}{270 \text{ miles}}$ or $\frac{270 \text{ miles}}{9.0 \text{ gallons}}$

7) $\$24 = 6 \text{ hours}$ or $\frac{\$24}{6 \text{ hours}}$ or $\frac{6 \text{ hours}}{\$24}$

Solution - Solving Problems

Example 1:

How many miles in 2,640 yards?

$$? \text{ miles} = 2{,}640 \text{ yd} \times \frac{3 \text{ ft}}{1 \text{ yd}} \times \frac{1 \text{ mile}}{5{,}280 \text{ ft}}$$

$$= \frac{2{,}640 \times 3}{5{,}280} \text{ miles}$$

$$= 1.500 \text{ miles}$$

2,640 yards = 1.500 miles

Solution - Solving Problems

Example 2:

What is the cost of three shirts, if a box containing 12 shirts costs \$27?

$$\$? = 3\ \cancel{\text{shirts}} \times \frac{\$27}{12\ \cancel{\text{shirts}}}$$

$$= \frac{3 \times \$27}{12}$$

$$= \$6.75$$

$$\boxed{\$6.75 = 3\ \text{shirts}}$$

Solution - Solving Problems

Example 3:

What is the gas consumption in miles per gallon of an automobile if it uses 0.1 gallons of gas in 100 s when traveling 60 miles/hr?

$$? \frac{\text{miles}}{\text{gallon}} = \frac{60 \text{ miles}}{1 \cancel{\text{hr}}} \times \frac{1 \cancel{\text{hr}}}{60 \cancel{\text{min}}} \times \frac{1 \cancel{\text{min}}}{60 \cancel{\text{s}}} \times \frac{100 \cancel{\text{s}}}{0.1 \text{ gallon}}$$

$$= \frac{60 \times 100}{60 \times 60 \times 0.1} \frac{\text{miles}}{\text{gallon}}$$

$$= \boxed{16.7 \frac{\text{miles}}{\text{gallon}}}$$

Solution - Solving Problems

Example 4:

How many grams HCl are in 3.800 mol HCl? The molar mass of HCl is 36.46 g/mol.

$$? \text{ g HCl} = 3.800 \cancel{\text{ mol HCl}} \times \frac{36.46 \text{ g HCl}}{1 \cancel{\text{ mol HCl}}}$$

$$= 3.800 \times 36.46 \text{ g HCl}$$

$$= 138.5 \text{ g HCl}$$

$$\boxed{138.5 \text{ g HCl} = 3.800 \text{ mol HCl}}$$

Solution - Solving Problems

Example 5:

How many moles NH_3 are in 5.28 g NH_3? (The molar mass of NH_3 is 17.031 g/mol.)

$$? \text{ mol } NH_3 = 5.28 \cancel{\text{ g } NH_3} \times \frac{1 \text{ mol } NH_3}{17.031 \cancel{\text{ g } NH_3}}$$

$$= \frac{5.28}{17.031} \text{ mol } NH_3$$

$$= 0.310 \text{ mol } NH_3$$

$$\boxed{0.310 \text{ mol } NH_3 = 5.28 \text{ g } NH_3}$$

Section III Solutions

8) $? \text{ s} = 800 \text{ min} \times \frac{60 \text{ s}}{1 \text{ min}} = \boxed{48000 \text{ s}}$

9) $? \text{ dozen of eggs} = 3500 \text{ eggs} \times \frac{1 \text{ dozen}}{12 \text{ eggs}} = \boxed{291.7 \text{ dozen of eggs}}$

10) $? \text{ mol } CO_2 = 360 \text{ g } CO_2 \times \frac{1 \text{ mol } CO_2}{44.01 \text{ g } CO_2} = \boxed{8.18 \text{ mol } CO_2}$

11) $? \text{ g OH} = 0.4300 \text{ mol OH} \times \frac{17.008 \text{ g OH}}{1 \text{ mol OH}} = \boxed{7.313 \text{ g OH}}$