B9 - Writing Numbers in Exponential Notation

Science Learning Center

Purpose and Objective

This learning module consists of a review of standard exponential notation. After completing this module, you should be able to:

- Write large and small numbers in exponential notation
- Convert numbers already in exponential form into standard exponential notation

Some rules will be presented to aid in the conversion of a number from one form to another.

Introduction

In science, as well as other disciplines, it is often necessary to work with extremely large and/or small numbers, such as the following:

Speed of Light = 30,000,000,000 cm/sec

Number of Atoms in 1 Gram of Carbon = 50,150,000,000,000,000,000,000 atoms

Mass of One Hydrogen Atom = 0.000000000000000000000001674 gram

Solubility of Cadmium Sulfide Gas in Water = 0.000000000000883 mole/liter

Introduction

A good way to avoid any issues with writing and working with large and small numbers is to express them in standard exponential notation or standard scientific notation as shown below:

Speed of Light: 3.00×10^{10} cm/sec

Number of Atoms in 1 Gram of Carbon: 5.015×10^{22} atoms

Mass of One Hydrogen Atom: 1.674×10^{-24} gram

Solubility of Cadmium Sulfide Gas in Water: 8.83×10^{-14} mole/liter

From just observing the space occupied by these numbers now, you can see that it is more efficient to work with these numbers.

Where Standard Exponential Notation Comes From

In order to make use of exponential notation, you must understand the derivation of both large and small numbers written in standard exponential notation. Consider the example below.

2173.0 is also equivalent to writing 2.1730×1000

or $2.1730 \times (10)(10)(10)$

and since $(10)(10)(10)=10^3$

we may also write 2.1730×10^3

This large number is now in standard exponential notation.

Where Standard Exponential Notation Comes From

In this example, we use the same process as the previous problem except instead of multiplying by a large number (1000 or 10^3), we are now multiplying by a small number, 0.0001 or 10^{-4}.

0.000716 is also equivalent to writing 7.16 x 0.0001

or 7.16 x (0.1)(0.1)(0.1)(0.1)

and since $0.1 = 10^{-1}$, and $(10^{-1})(10^{-1})(10^{-1})(10^{-1}) = 10^{-4}$

we may also write 7.16 x 10^{-4}

This small number is now in standard exponential notation.

Where Standard Exponential Notation Comes From

The method of conversion to standard exponential notation is not the easiest method to use, but it does give you an idea of where an exponential comes from.

An easier method is on the next page.

Rules for Writing Standard Exponential Notation

From the previous two examples, a general rule can be seen. In standard exponential notation, the number is always written with only **one digit** to the left of the decimal.

Example:

The correct way to write 2,173 in standard exponential form is $\mathbf{2.173 \times 10^3}$

The number 2,173 can also be written in a non-standard notation as the following:

$$2{,}173 = 21.73 \times 10^2$$
$$2{,}173 = 217.3 \times 10^1$$
$$2{,}173 = 2173 \times 10^0$$
$$2{,}173 = 0.2173 \times 10^4$$
$$2{,}173 = 0.02173 \times 10^5$$

The above ways are not standard because only one digit should be to the left of the decimal.

Rules for Writing Standard Exponential Notation

When converting the number 2173.0 to standard exponential notation, we have to move the decimal 3 places to the left.

Another general rule can be derived from this example: The exponent of 10 on a large number is equal to the number of places the decimal point has to be moved.

Example:

$2173.0 = 2.1730 \times 10^{3}$

Decimal moves 3 places to the left, therefore the exponent on 10 is 3

Rules for Writing Standard Exponential Notation

Another Example:

$3{,}170{,}000{,}000 = 3.17 \times 10^{9}$

Decimal moves 9 places to the left

(Exponent on 10 is 9)

Rules for Writing Standard Exponential Notation

When working with small numbers, the same general pattern continues. Below, the example 0.000716 is worked out. However, as a general rule, the exponent of a small number is a negative number equal to the number of places the decimal point moves.

$$0.000716 = 7.16 \times 10^{-4}$$

Decimal moves 4 places to the right

(Exponent on 10 is 4 with a negative sign)

Rules for Writing Standard Exponential Notation

Another example is given below:

$0.0000575 = 5.75 \times 10^{-5}$

Decimal moves 5 places to the right

(Exponent on 10 is 5, but with a negative sign)

Rules for Writing Standard Exponential Notation

The rules for writing a number in standard exponential notation will not change if the exponent is 0.

Example:

Suppose you have the number 5.134 and wish to write it in standard exponential notation. Your answer would be:

$$5.134 \times 10^0$$

Because $10^0 = 1$

And $(5.134 \times 10^0) = (5.134 \times 1) = 5.134$

Summary of Rules for Writing Standard Exponential Notation

In summary, to convert a number to standard exponential notation:

1. Move the decimal point, so that there is only one digit to the left of the decimal.
2. Count the number of places the decimal moves.
3. If the number is large (or the decimal moves to the left), the exponent on 10 is equal to the number of places the decimal was moved and has a positive sign.
4. If the number is small (or the decimal moves to the right), the exponent on 10 is equal to the number of places the decimal was moved and has a negative sign.

This process results in a number of the following general form: $C \times 10^{n}$

where C is the number between 1 and 9.999…

and $\boldsymbol{n}$ is a positive or a negative integer.

Practice Problem Set 1:

Before proceeding, convert the following numbers to standard exponential form on a piece of scratch paper.

1) 3,760,000,000
2) 0.0000567
3) 0.0000000091
4) 476,100

Solutions in the back of the module

Converting Exponential Numbers to Standard Form

Numbers that result from calculations are often *not* in standard exponential notation.

You will need to *change the value of the exponent* to have your resulting number in the correct standard form.

(You may also be able to set your calculator to display results in correct standard form).

Converting Exponential Numbers to Standard Form

Example:

$$\begin{array}{rl} & 9.46 \times 10^{2} \\ \times & 8.63 \times 10^{1} \\ \hline & 81.6 \times 10^{3} \end{array}$$

To convert to standard form:

$81.6 \times 10^{(3+1)}$

(1 is added to the current exponent of 10)

8.16×10^{4}

- Multiplying these two numbers gives an answer that is not in standard exponential form.
- To convert this number to standard exponential notation, we need to move the decimal point to the left and increase the exponent on 10 by 1.

Converting Exponential Numbers to Standard Form

Below is an example with a negative exponent of 10. We again add a number, equal to the number of places the decimal was moved to the left, to the exponent of 10.

2040.0×10^{-5}

Decimal moves 3 places to the left

$2.0400 \times 10^{(-5+3)}$

(3 is added to the current exponent of 10)

2.0400×10^{-2}

Converting Exponential Numbers to Standard Form

If the number is small and the decimal must move to the right, we subtract the number of places the decimal moves from the current exponent as shown in the following example.

0.0414×10^{-3}

Decimal moves 2 places to the right

$4.14 \times 10^{(-3-2)}$

(2 is subtracted from the current exponent of 10)

4.14×10^{-5}

Converting Exponential Numbers to Standard Form

Another example:

0.00051×10^{9}

Decimal moves 4 places to the right

$5.1 \times 10^{(9-4)}$

(4 is subtracted form the current exponent of 10)

5.1×10^{5}

Summary of Converting Exponential Numbers to Standard Form

We can formulate two general rules:

1) If the decimal point moves to the left (or the number is large), **add** the number of places the decimal moves to the current exponent of 10.

2) If the decimal point moves to the right (or the number is small), **subtract** the number of places the decimal moves from the current exponent of 10.

Practice Problem Set 2:

Convert the following numbers to standard exponential notation on a piece of scratch paper.

1) 0.00416×10^{6}

2) 24.8×10^{-3}

3) 0.716×10^{-4}

4) 3410×10^{2}

Check Your Work for Reasonableness!

Finally, you should always check calculations or conversions of any kind to be certain they appear reasonable. You can check these conversions by writing them out as regular numbers.

In the previous examples, you converted 0.0052×10^4 to 5.2×10^1. Write out both of these numbers. They should be equal if the conversion was done correctly.

$$0.0052 \times 10^4 = 52$$

Check!

$$5.2 \times 10^1 = 52.$$

Check Your Work for Reasonableness!

Another example:

$714.24 \times 10^7 = 7{,}142{,}400{,}000$

Check!

$7.1424 \text{ X } 10^9 = 7{,}142{,}400{,}000$

Final Directions

When you feel you understand the material presented in this module, obtain and complete a posttest. Have the test checked before you leave. If you make any mistakes in the posttest you may review this module and retake the test as many times as necessary.

Good Luck!

Answers to Practice Problem Set 1:

1. 3.76×10^{9}
2. 5.67×10^{-5}
3. 9.1×10^{-9}
4. 4.761×10^{5}

Answers to Practice Problem Set 2:

1. 4.16×10^{3}
2. 2.48×10^{-2}
3. 7.16×10^{-5}
4. 3.41×10^{5}